Single pulsar noise analysis

Siyuan Chen*
Shanghai Astronomical Observatory, CAS

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*Email: siyuan.chen@shao.ac.cn

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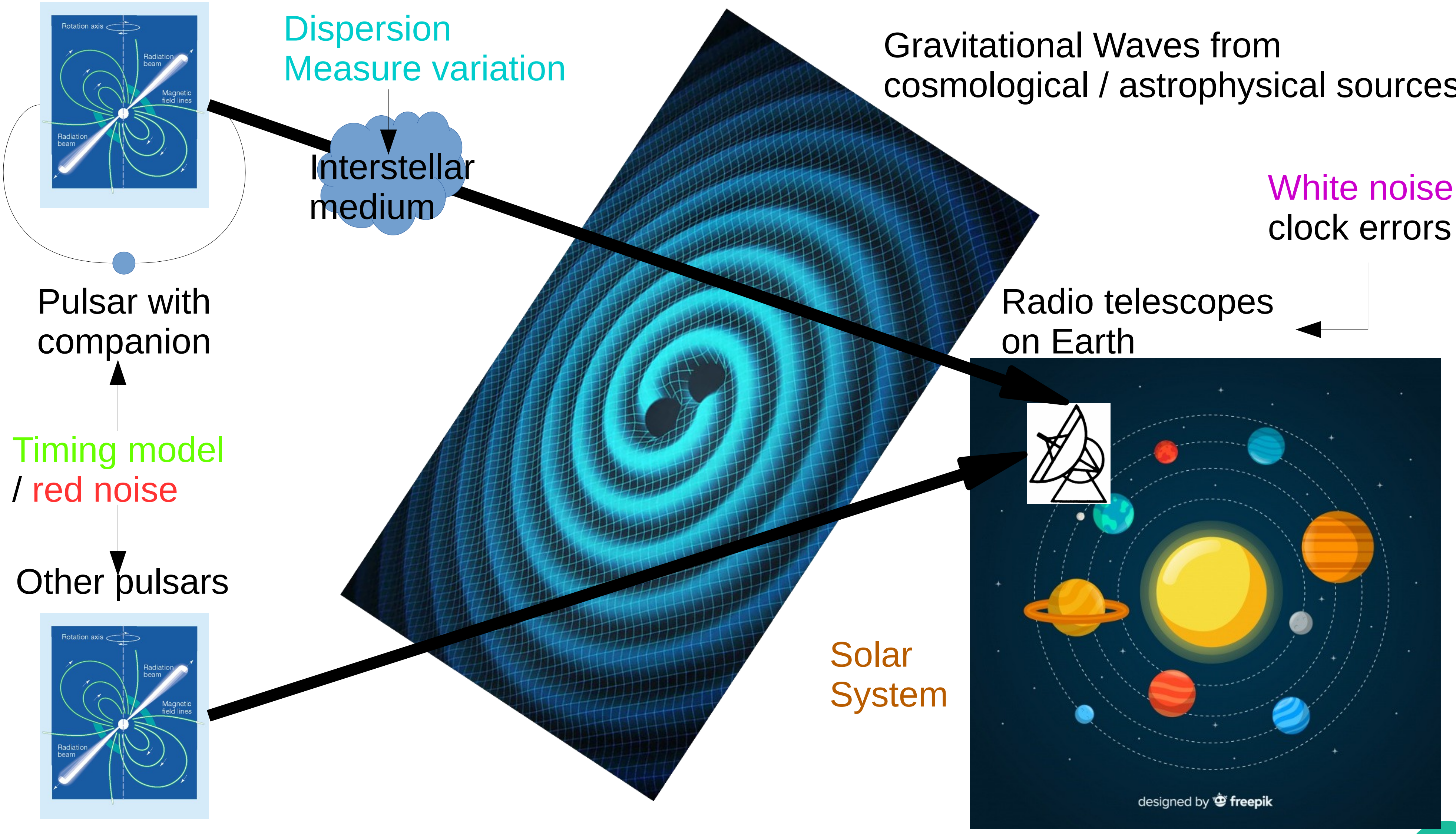
Signal and noise



Signal and noise

Model all noises/signals as Gaussian processes to simplify likelihood computation:

Note: timing model marginalized

$$p(\delta t|\phi) = \frac{\exp(-\frac{1}{2}\delta t^T \Sigma^{-1} \delta t)}{\sqrt{\det(2\pi\Sigma)}}$$

Note: solar system modelled separately

$$\Sigma^\alpha = \Sigma^\alpha_{WN} + \Sigma^\alpha_{RN} + \Sigma^\alpha_{DM} + \Sigma^{\alpha\beta}_{GW}$$

white noise

$$S_{WN} = E^2(\sigma^2 + Q^2) + C$$

red noise

$$S_{RN} = A^2_{RN} f^{-\gamma_{RN}}$$

dispersion measure

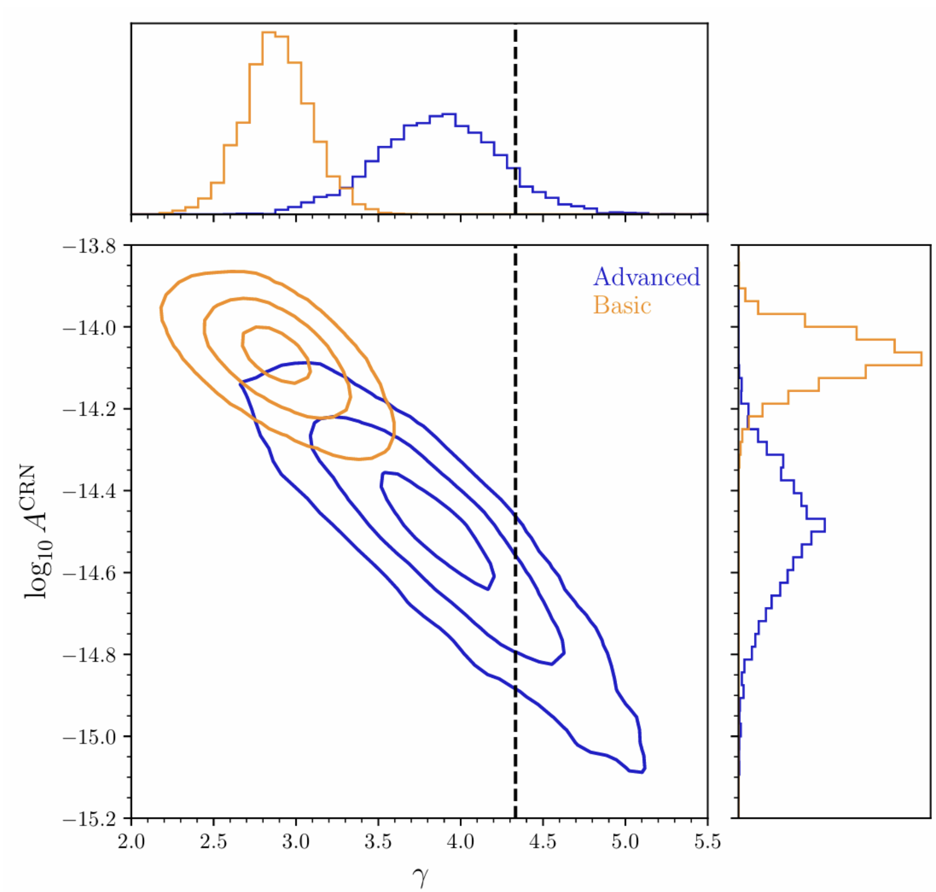
$$S_{DM} = \frac{K_{DM}}{\nu^2} A^2_{DM} f^{-\gamma_{DM}}$$

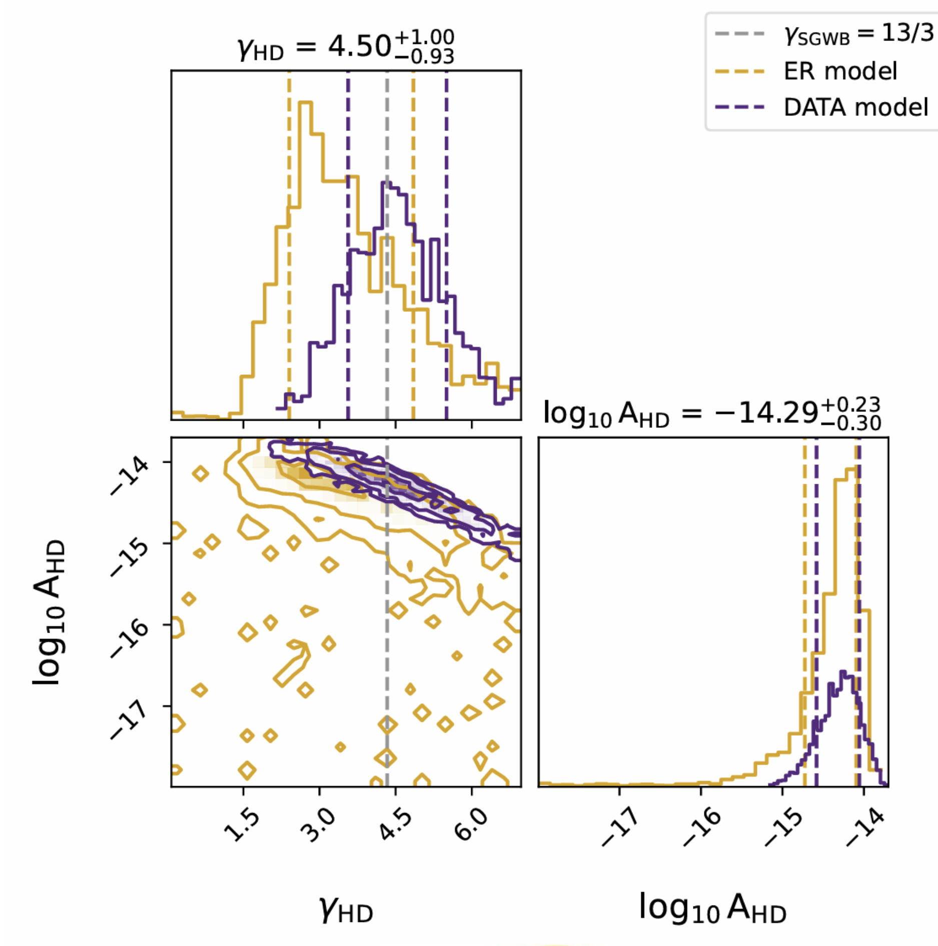
gravitational wave background

$$S_{GW} = \Gamma_{\alpha\beta} A^2_{GW} f^{-\gamma_{GW}}$$

Van Haasteren and Vallisneri 2014

Gravitational wave background - posterior constraints



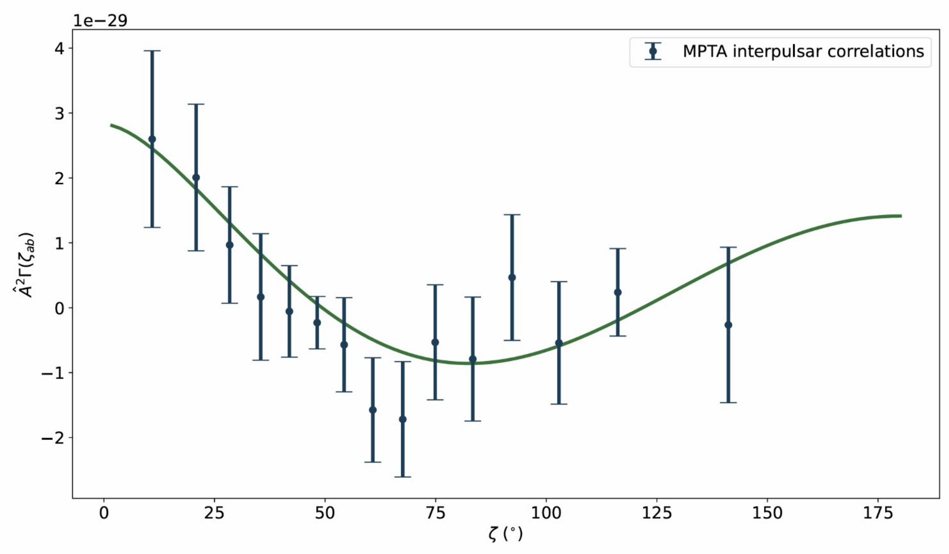


Arxiv: 2306.16229, 2412.01153

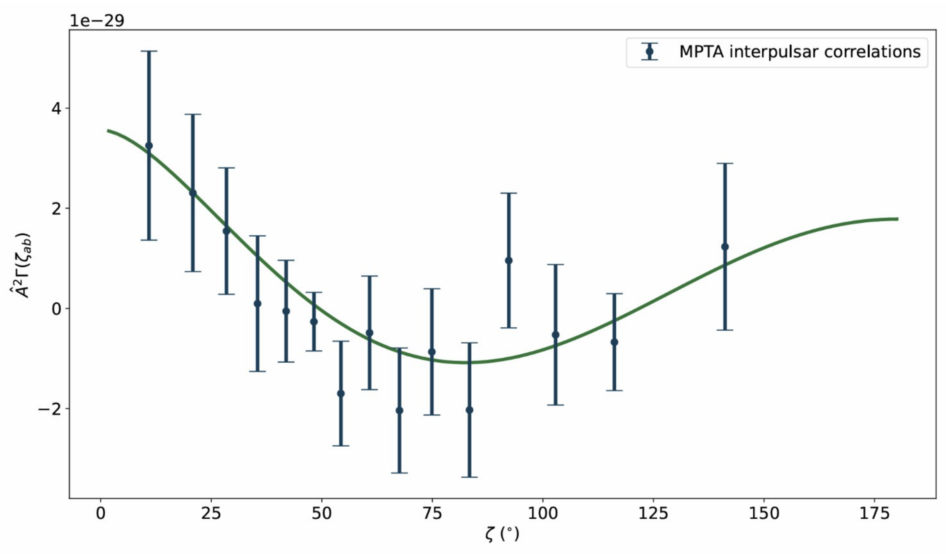
PPTA MeerKAT

Gravitational wave background - spatial correlation

DATA model: ~3 sigma



ALT model: ~4 sigma

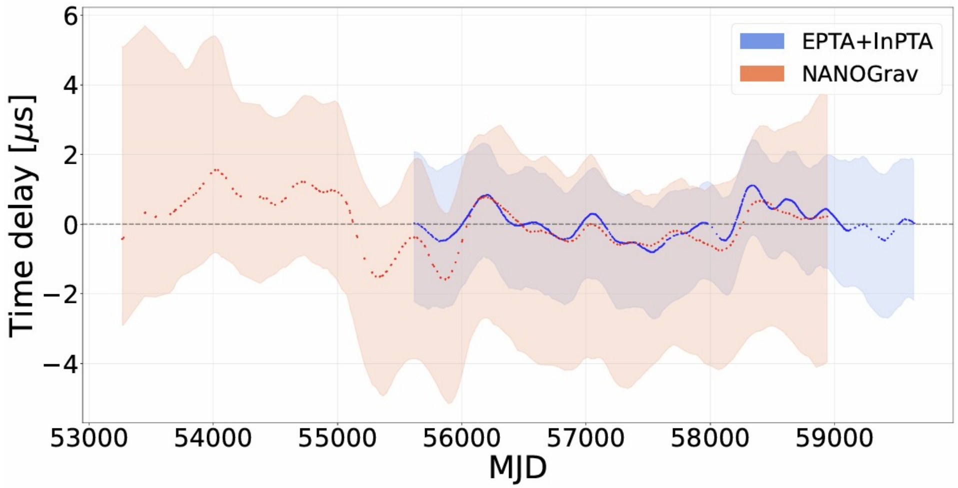


Arxiv: 2412.01153

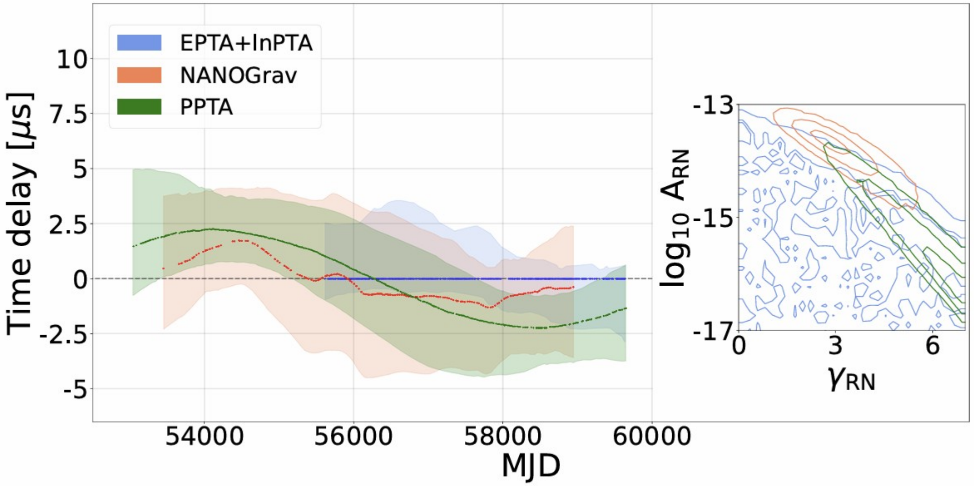
MeerKAT

IPTA comparison

J012+5307

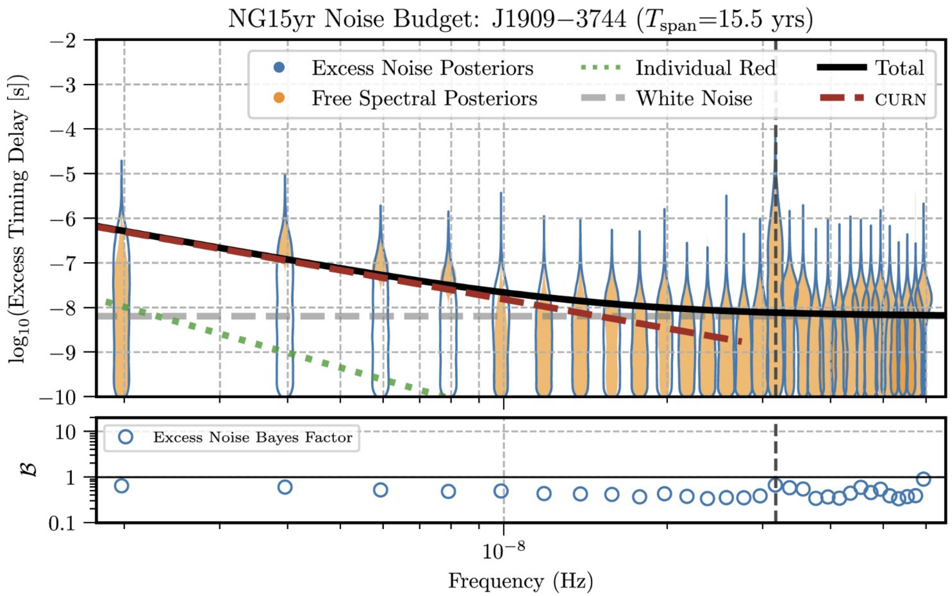


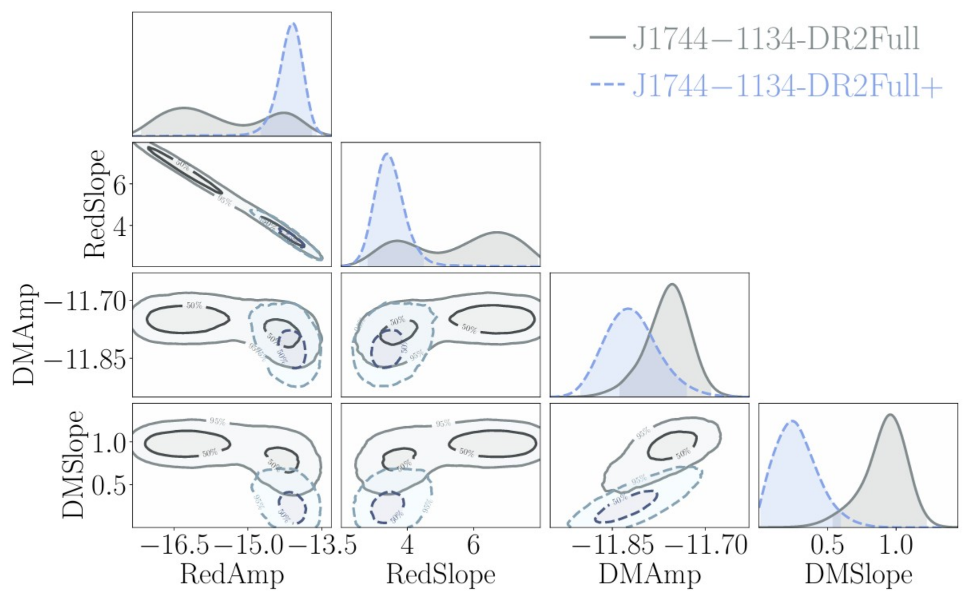
J0613-0200



Arxiv: 2309.00693

Pulsar noise constraints





Arxiv: 2306.16219, 2306.16227

Noise terms as time correlated signal

Covariance matrix

Timing residuals j and k

PSD in the Fourier frequency f domain with dependence on the radio observation frequency nu

$$\mathrm{C}_{\mathrm{X},jk} = \langle \delta t_{\mathrm{X},j}\, \delta t_{\mathrm{X},k} \rangle = \int_0^\infty S_{\mathrm{X}}(f,\nu) e^{2\pi i f (t_j - t_k)} df \,,$$

Noise terms as time correlated signal

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Achromatic red noise (RN), independent of nu; also used for system and band noise

$$S_{RN}(f) = \frac{A_{RN}^2}{12\pi^2} \left(\frac{f}{f_{yr}} \right)^{-\gamma_{RN}} \mathrm{yr}^3$$

Chromatic noise, dependent on nu; for amplitude DM: -2, scattering: -4, or free index

$$S_{DM}(f,\nu) = \frac{A_{DM,TN}^2}{K^2 \nu^4} \left(\frac{f}{f_{yr}} \right)^{-\gamma_{DM}} \mathrm{yr}^3$$

Relation between DM residuals and timing residuals via nu^2

$$\delta DM_{Y,j} = K \nu_j^2 \delta t_{DM_{Y},j}$$

CPTA, Chen et al., 2025

White noise

tempo, tempo2, PINT convention:

Kronecker delta jk:
delta jk = 1, if j=k
delta jk = 0, otherwise

$$C_{\mathrm{WN},jk} = \mathrm{EFAC}^2(\sigma_j\sigma_k + \mathrm{EQUAD}^2)\delta_{jk} + \mathrm{ECORR}^2 C_{\mathrm{ECORR},jk}$$

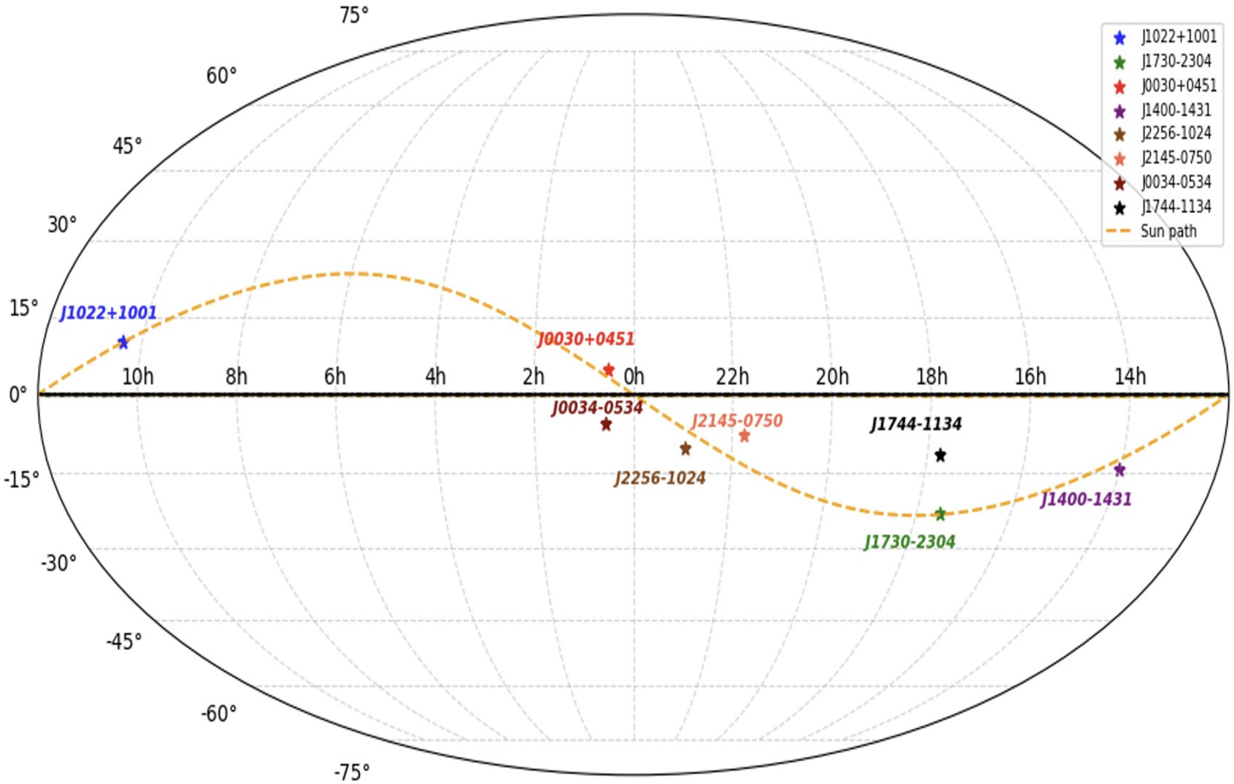
temponest convention:

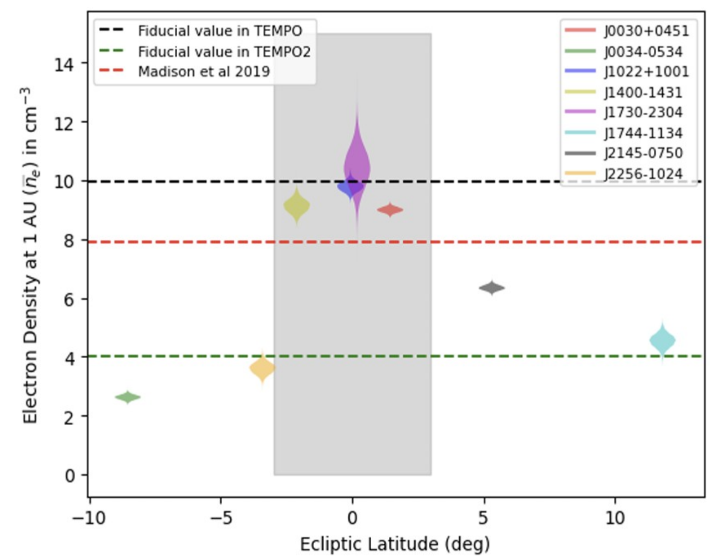
$$C_{\mathrm{WN},jk} = \mathrm{EFAC}^2\sigma_j\sigma_k\delta_{jk} + \mathrm{EQUAD}^2\delta_{jk} + \mathrm{ECORR}^2 C_{\mathrm{ECORR},jk}$$

- One EFAC, EQUAD (and ECORR) per receiver+backend system
- Sometimes global parameters or across several systems
- Usually, hundreds of parameters, thus fixed for the GW searches

C ECORR = 1, if j and k are from the same observation
C ECORR = 0, otherwise

Solar wind





$$DM_{\rm SW} = n_e \frac{\rho}{r_e \sin\rho} [1\ {\rm AU}]^2$$

$$S_{\rm SW} = A^2_{\rm SW} \left(\frac{f}{{\rm yr}^{-1}}\right)^{-\gamma_{\rm SW}} {\rm yr}^3,$$

Susarla et al., 2024, Iraci et al., 2025
(EPTA+LOFAR+NenuFAR)

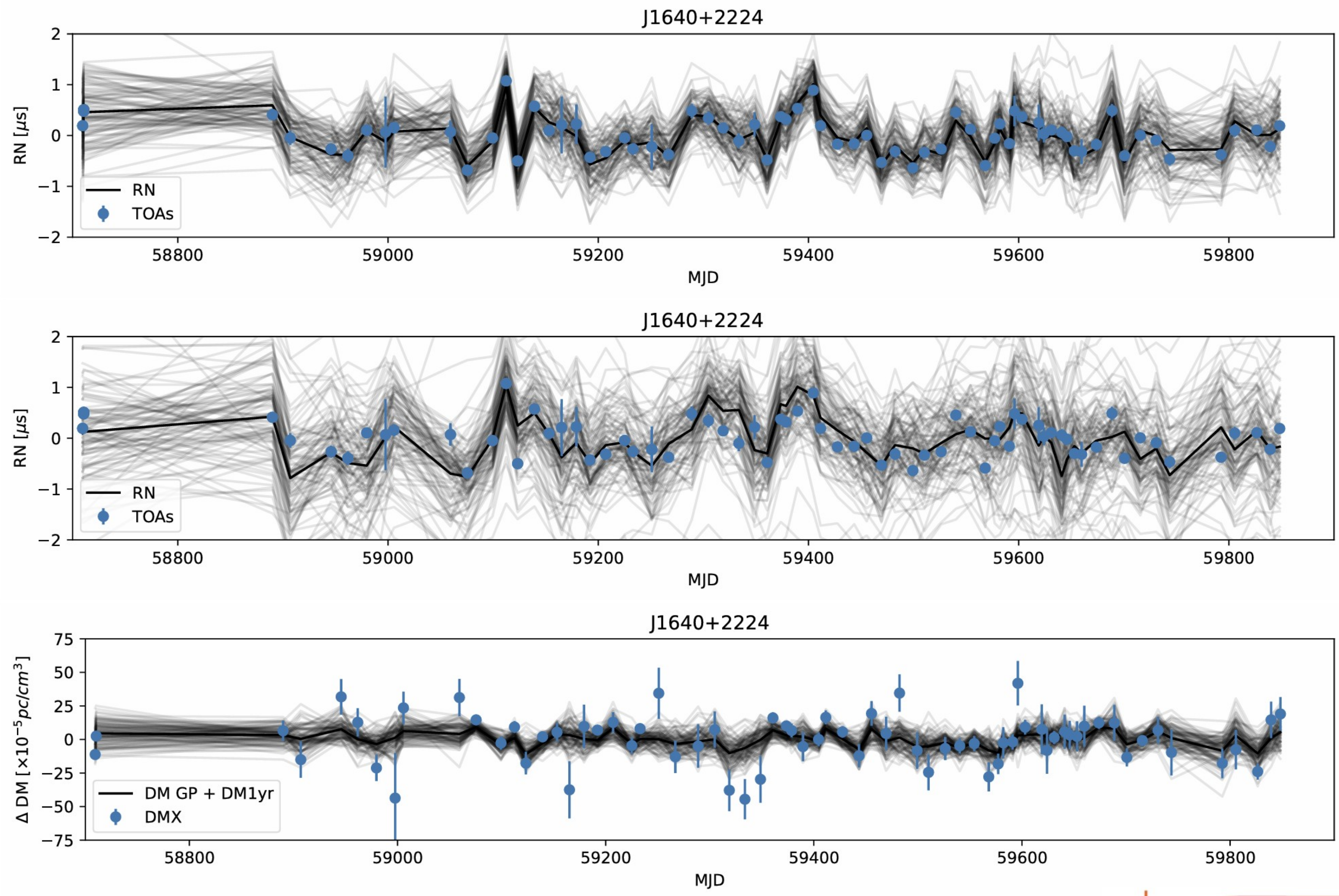
Number of parameters

Pulsar timing model ~ 15 parameters + number of systems	Period, period derivatives, position, proper motion, binary parameters, system offsets, etc.
Pulsar intrinsic noise (red noise) >2 parameters	Amplitude and spectral index, excess noise
Interstellar medium (DM noise) >2 parameters	Amplitude and spectral index, deterministic events/variations
White noise ~ 20 systems	2 or 3 parameters per system
Global white noise	Few parameters

Extra noise terms	2 or 3 parameters per noise term
System noise 2 or 3 parameters per backend system	Instrumental red noise per backend system
Band noise 2 or 3 parameters per radio frequency band	Instrumental red noise per radio frequency band
Chromatic noise (Scattering) >2 parameters	Amplitude and spectral index
Solar wind >2 parameters per pulsar (global)	Distance to the ecliptic of the Sun; global vs per pulsar

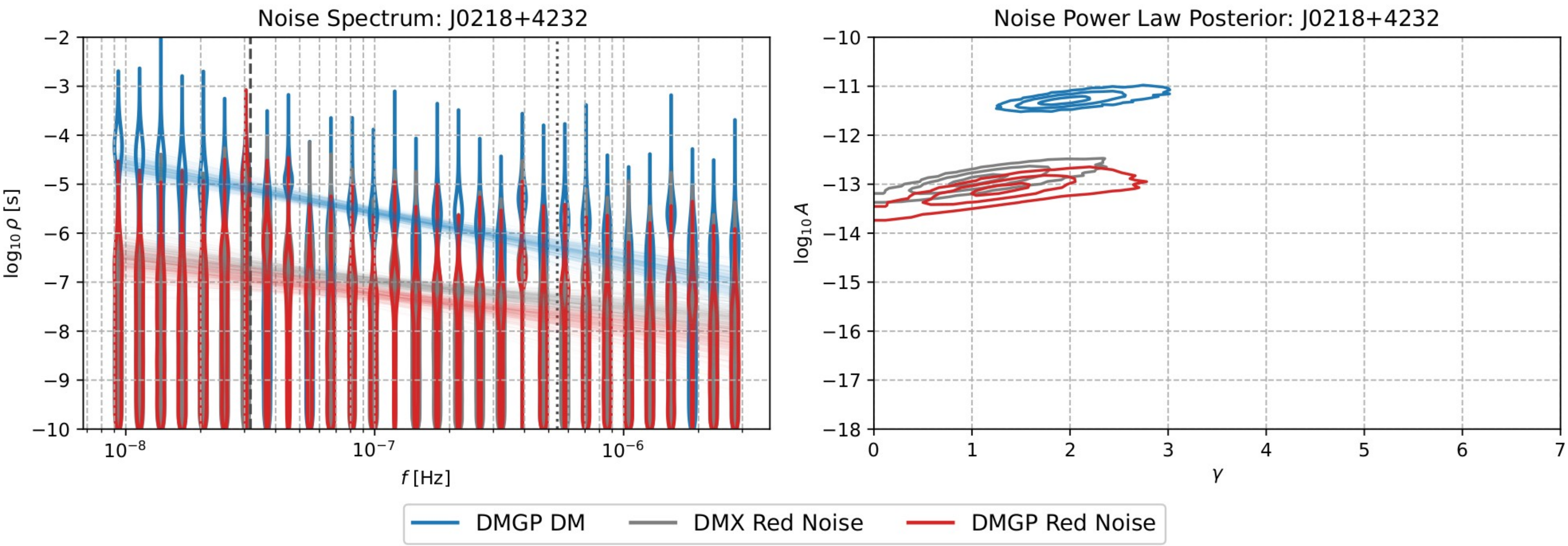
~100 parameters per pulsar; some can be marginalized, others can use fixed values

Pulsar noise realizations



CPTA DR1 single pulsar noise analysis, arxiv: 2506.04850

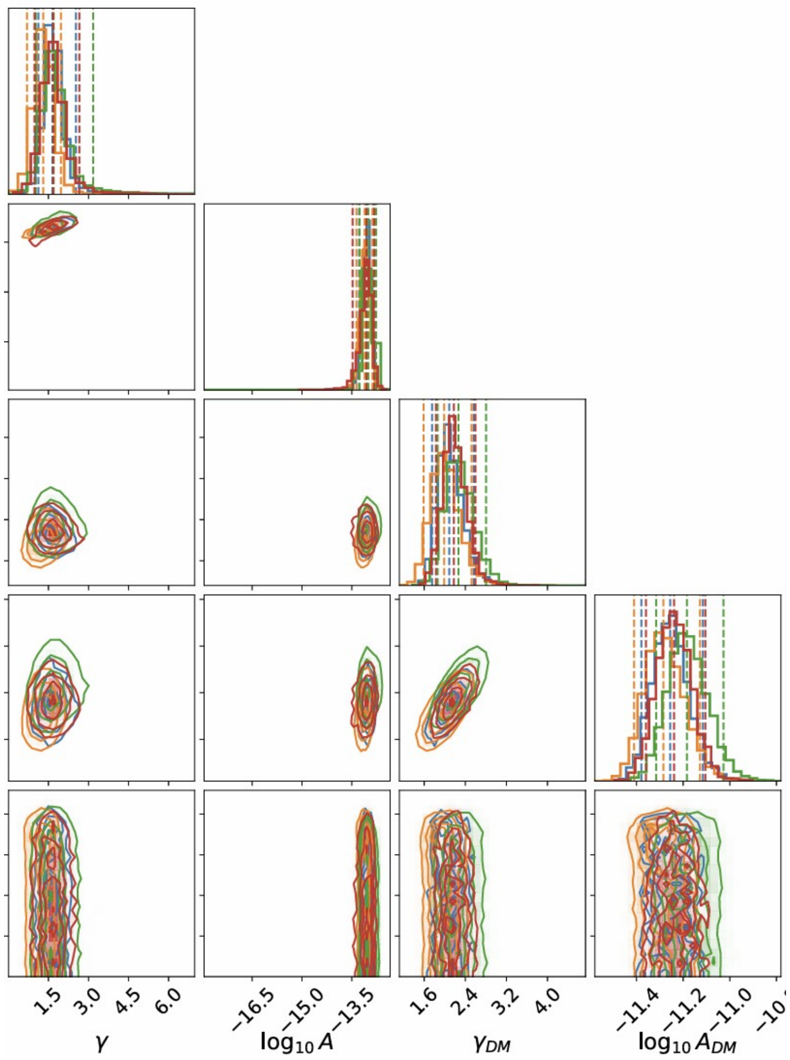
Choice of frequency binning

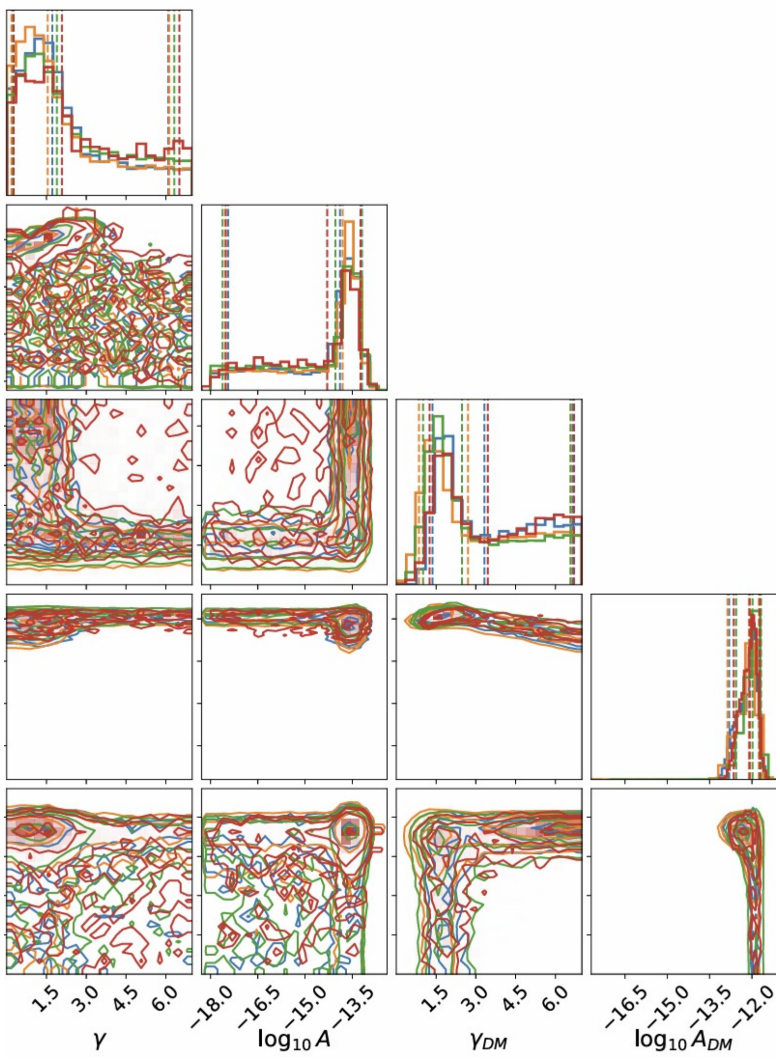


$$S_{RN} = \frac{A^2}{12\pi^2}\left(\frac{f}{f_{\rm yr}}\right)^{-\gamma}\ \mathrm{yr}^3$$

Optimize number of frequency bins and high frequency cutoff based on highest Bayes factors and stable posteriors

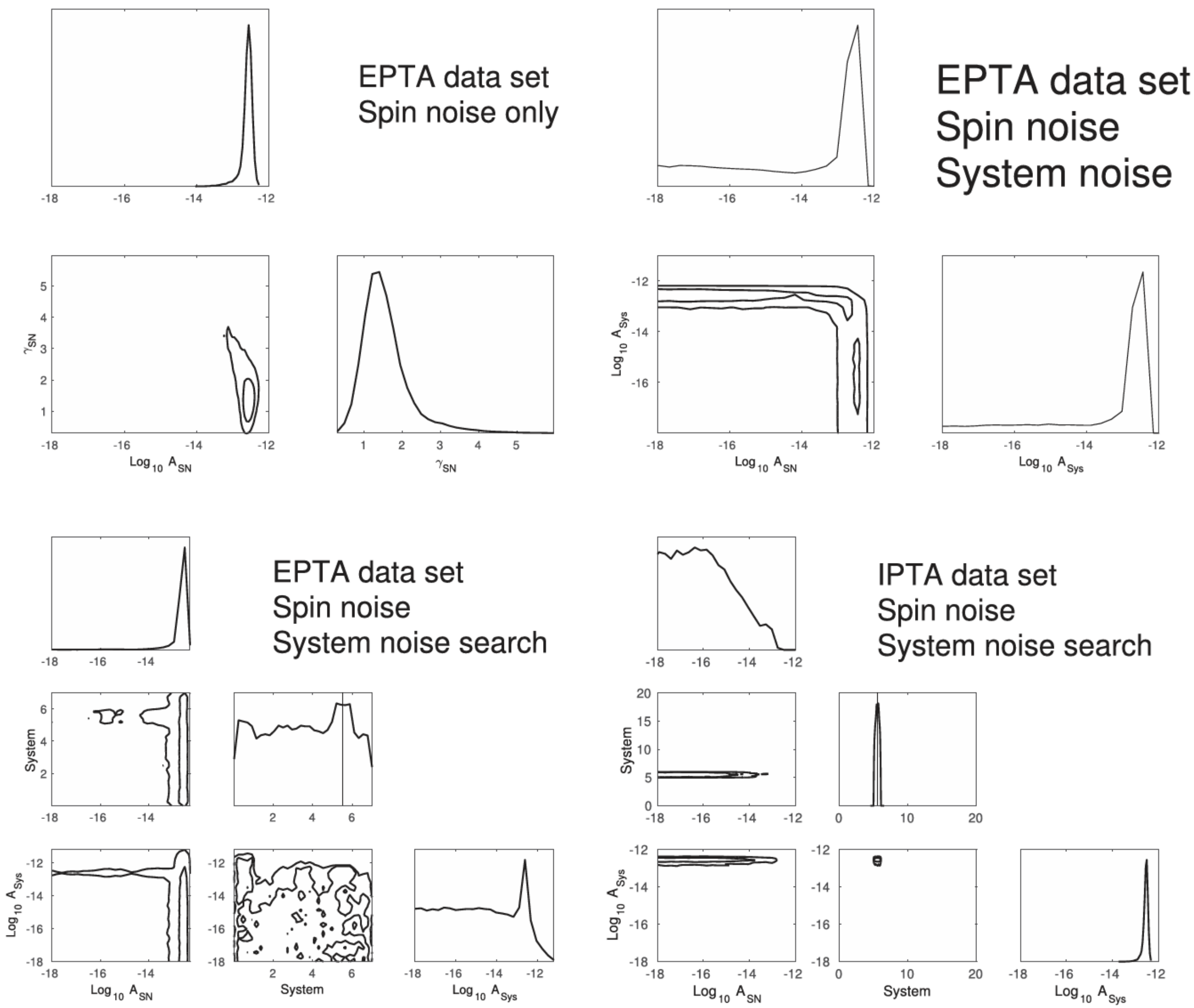
Posterior distributions





IPTA DR1 noise analysis

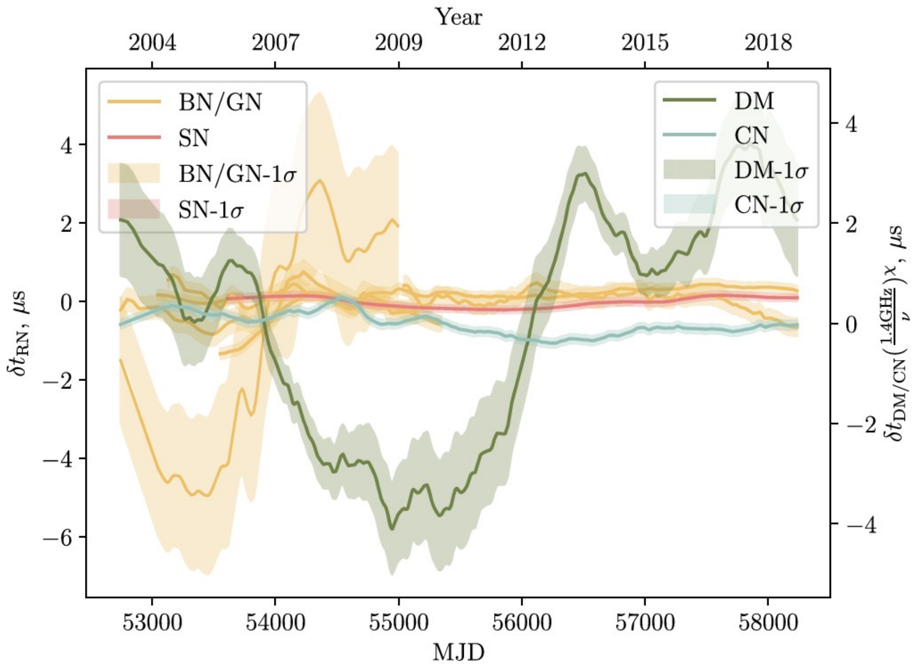
J1730-2304



IPTA, Lentati et al., 2016

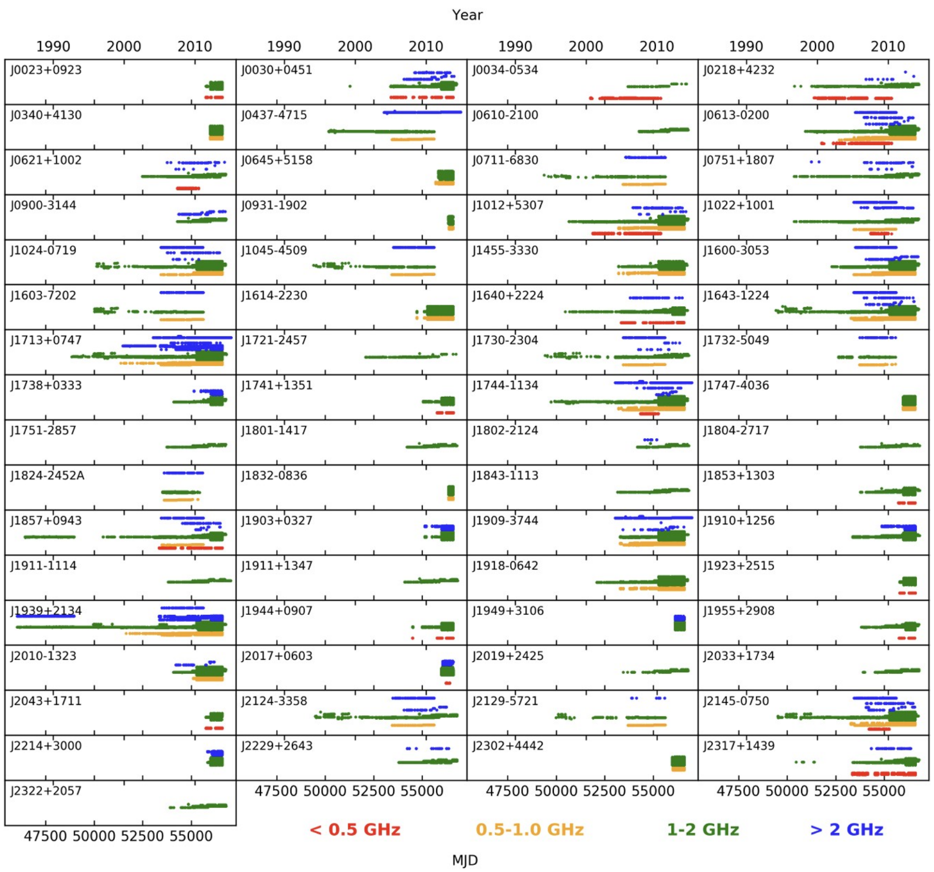
System and band noise

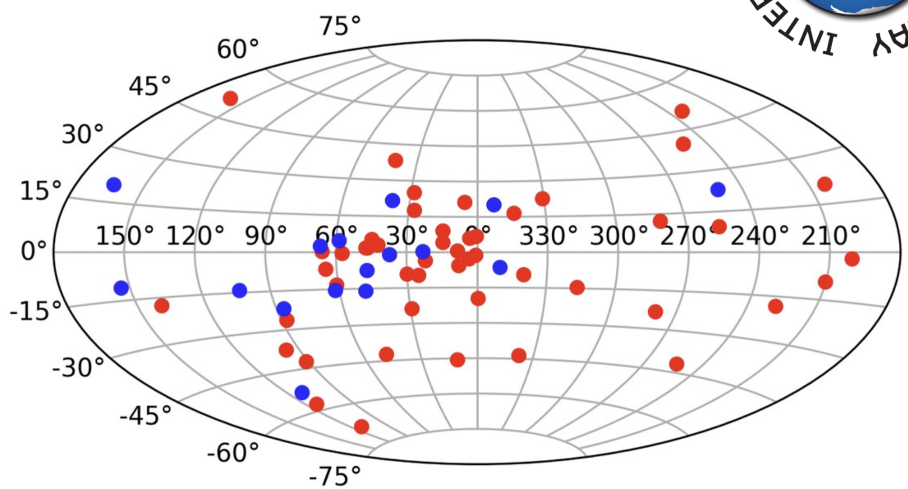
PSR	Flag and value	$\log_{10}A$	γ	$\ln\mathcal{B}$	n_c
J0437−4715	-group CPSR2_10CM	$-13.40^{+0.09}_{-0.10}$	$1.13^{+0.38}_{-0.44}$	156.9	36
	-group CPSR2_20CM	$-13.28^{+0.11}_{-0.07}$	$1.18^{+0.54}_{-0.34}$	284.9	39
	-group WBCORR_10CM	$-13.47^{+0.19}_{-0.10}$	$0.41^{*+0.57}_{-0.30}$	85.9	10
	-group CPSR2_50CM	$-12.85^{+0.08}_{-0.08}$	$2.43^{+0.47}_{-0.24}$	323.6	36
	-B 40CM -B 50CM	$-13.51^{+0.10}_{-0.16}$	$1.88^{+0.87}_{-0.20}$	356.8	90
	-B 20CM	$-13.80^{+0.08}_{-0.07}$	$2.90^{+0.43}_{-0.23}$	353.8	90
J1600−3053	-B 40CM -B 50CM	$-12.61^{+0.08}_{-0.10}$	$1.85^{+0.36}_{-0.28}$	27.9	86
J1643−1224	-B 40CM -B 50CM	$-12.06^{+0.04}_{-0.04}$	$2.33^{+0.30}_{-0.19}$	105.4	85
J1713+0747	-B 10CM -B 20CM	$-14.49^{+0.23}_{-0.65}$	$4.18^{+1.22}_{-0.91}$	4.4	86
	-group CPSR2_20CM	$-13.57^{+0.21}_{-0.19}$	$1.60^{+1.35}_{-0.22}$	21.9	36
J1744−1134	-B 10CM -B 20CM	$-13.56^{+0.17}_{-0.02}$	$1.00^{+0.54}_{-0.20}$	12.7	86
J1824−2452A	-B 40CM -B 50CM	$-12.06^{+0.07}_{-0.06}$	$1.01^{+0.32}_{-0.19}$	78.2	83
J1909−3744	-B 40CM -B 50CM	$-13.42^{+0.10}_{-0.08}$	$0.64^{+0.47}_{-0.21}$	62.3	85
J1939+2134	-B 40CM -B 50CM	$-13.16^{+0.10}_{-3.29}$	$3.13^{*,+4.33}_{-1.57}$	135.1	84
J2124−3358	-B 20CM	$-16.07^{+1.81}_{-0.87}$	$8.40^{+0.56}_{-4.26}$	1.9	84
J2145−0750	-group CPSR2_50CM	$-14.74^{+1.14}_{-0.43}$	$7.18^{*,+1.97}_{-2.60}$	16.9	65



PPTA DR2, Goncharov et al., 2021
E.g., J0437-4715, various noise terms present in the data set

IPTA data combination





IPTA DR2, Perera et al., 2019
IPTA DR3, ongoing, 110+ pulsars, 30+ observing systems

Summary

- Standard noise model: RN, DM, white noise; white noise fixed for GW search
- Customized noise models developped over the years: varied frequency binning, system noise, band noise, chromatic noise, solar wind
- Customized noise model may be required to detect and characterize GW signals
- IPTA combinations will become more and more challenging with more data, more pulsars and more complex noise models
- Problem / solution?